Kinematics: First Impression

Questions

1. Rotation Matrices Turned Around **[10%]**

Given the following 3x3 rotation matrices:

$$R_1 = \begin{bmatrix} 0.7071 & -0.7071 & 0 \\ 0.7071 & 0.7071 & 0 \\ 0 & 0 & 1 \end{bmatrix}, R_2 = \begin{bmatrix} 0.6124 & -0.5 & 0.6124 \\ 0.3536 & 0.866 & 0.3536 \\ -0.7071 & 0 & 0.7071 \end{bmatrix}$$

$$R_3 = \begin{bmatrix} 0.6124 & 0.6124 & 0.5 \\ 0.3536 & 0.3536 & -0.866 \\ -0.7071 & -0.7071 & 0 \end{bmatrix}, R_4 = \begin{bmatrix} 0 & 0.2588 & 0.9659 \\ 0 & 0.9659 & -0.2588 \\ -1 & 0 & 0 \end{bmatrix}$$

$$R_5 = \begin{bmatrix} -0.9122 & 0.4098 & 0 \\ -0.4098 & -0.9122 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$R_7 = \begin{bmatrix} 0.8889 & 0.1111 & 0.6667 \\ 0.3333 & 0.5556 & 0.7778 \\ 0.4444 & 1 & 0.2222 \end{bmatrix}, R_8 = \begin{bmatrix} 0.1191 & -0.9201 & 0.3731 \\ 0.8592 & -0.0927 & -0.5031 \\ 0.4975 & 0.3805 & 0.7795 \end{bmatrix}$$

a. Are these (within practical numerical limits) valid rotation matrices? Why?

b. If yes, determine the Roll, Pitch, and Yaw that define each matrix. Please state your assumptions. Do you believe their values?

2. A camera with attitude **[10%]**

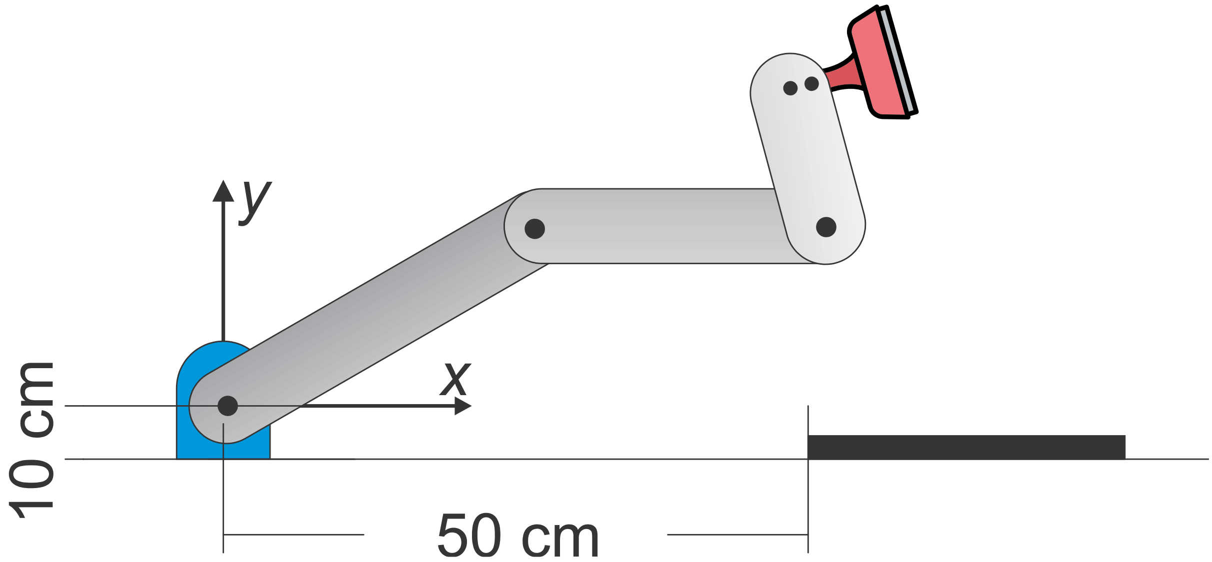
A camera has its *z*-axis parallel to the vector [0, 1, 0] in the world frame, and its *y*-axis parallel to the vector [0, 0, −1]. What is the orientation of the camera with respect to the world frame expressed as:

a. a rotation matrix?

b. a unit quaternion?

3. A Stamp Duty **[30%]**

A small humanoid robot is being programmed to duly stamp to a form (for which a duty is collected). The first objective is to place the stamp on a form placed in the 420 mm (A3) wide paper tray (solid back outline) in the figure below. Assume that the arm is composed of 3 revolute joints and is constrained to move in the plane of the page. The arm consists of 3 links with joint-to-joint dimensions: L_1=0.5m, L_2=0.25m, L_3=0.15 m (to the stamp rivets). The stamp is riveted orthogonally to the last link (L_3) and is 10 cm in extension from the bottom of rivet location. The stamp must be within 15 degrees of paper for a correct mark to be placed.



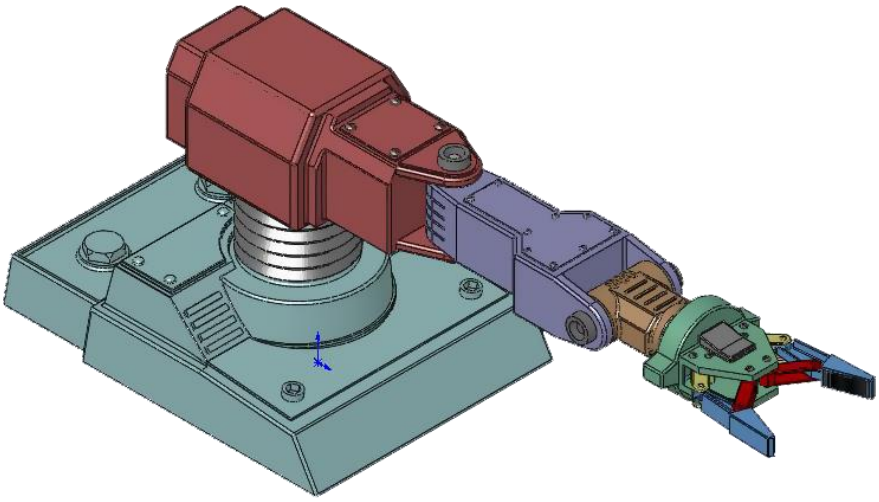
Please calculate valid workspace and joint positions for the final stamp location. **Then**, assuming initial arm angles of $\theta_1=0^o$, $\theta_2=15^o$, $\theta_3=30^o$, kindly calculate trajectory function so as to place the stamp correctly on the form regardless of where it is placed in the tray.

Mark the Stamp. Then use this to plot a valid trajectory for a form at the center of the tray.

A Riveting Conclusion. Please modify the trajectory function from above so to allow for placement stamp if the lower rivet holding the stamp in place breaks so that the stamp itself may rotate freely.

4. Back to the Future -- Forward & Differential Kinematics **[30%]**

Consider the Armatron robot shown in the figure below

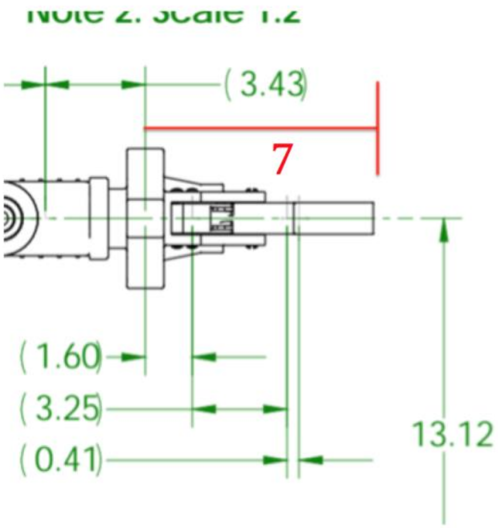


(See links for enlarged views of the arm [**png**], solid model [**stl**], and **its dimensions** [**pdf**])

a. Please form a kinematic diagram (refer to tutorial 3, question 1) of the arm, including link lengths.

b. Adopting the "Standard" Denavit-Harenberg (D-H) convention, what are the D-H parameters/table for the robot? (i.e., for each link (i), the parameters a_i, α_i, d_i, θ_i)

c. For this arm, please derive the forward kinematics for the centre (tip when closed) of the end-effector (the blue part in the above isometric CAD view) given the length of the gripper is 7cm as shown below.



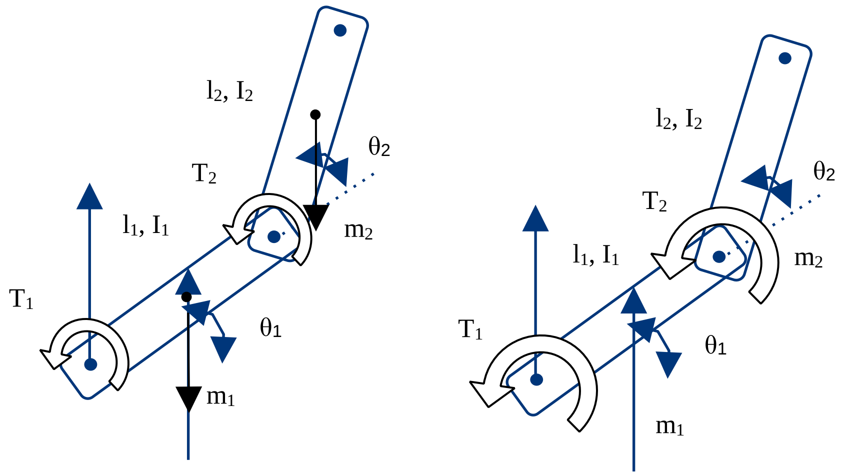
d. What is the angular velocity of the tip of the end-effector in terms of the joint rates? (Please express this in a fixed frame at the origin).

5. Gravity! **[20%]**

Well beyond popular culture, gravity is a subject of deep space and thought. It is also a critically important subject in robotics, such as for robot arm dynamics. Consider the two link manipulator below in both a vertical and horizontal configuration (relative to gravity).

(a) vertical configuration (gravity ↓)

(b) horizontal configuration (gravity into page □)



l_1 = 1 m
m_1 = 5 kg
I_1 = 0.4 kgm^2
θ_1 = joint angle 1
$\boldsymbol{\tau}_1$ = torque at joint 1

l_2 = 0.75m
m_2 = 4 kg
I_2 = 0.2 kgm^2
θ_2 = joint angle 2
$\boldsymbol{\tau}_2$ = torque at joint 2

a. Derive the equations of motion that describe the system dynamics of the two configurations.

b. For a given torque-limited motor (**τ^*_{max}**), what is the maximum load (weight) the robot can move in each (i.e., vertical and horizontal) configuration?

c. Based on the results above, please describe the effect of gravity on robot arm dynamics.

Due Date

The problem set must be completed **individually** by the end of Week 5 (23:59 Friday**, August 23, 2019**). Submission is via Platypus.

Early submission is **highly** encouraged.